Swami Ramanand Teerth Marathwada University, Nanded, M.S., India-431606

(*UGC Recognized, NAAC Re-accredited 'B++'grade state university*)

Study of Some Fractional-order Generalized Thermoelastic Problems and its Applications Using Computational Software's

Research Proposal submitted

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In the subject

Mathematics faculty of Science and Technology

Submitted by

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Synopsis

Name of the student: **Mrs. Sandhya Uttamrao Lokhande**

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Thesis title: **Study of Some Fractional-order Generalized Thermoelastic Problems and its Applications Using Computational Software's**

Thesis supervisors: **Dr. Pramod R. Kulkarni**

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Abstract of proposed research work:

1. Introduction

Thermoelasticity is a branch of physics that deals with the study of the mechanical behavior of materials under the combined effects of thermal and mechanical loads. It explores the relationship between temperature changes and the resulting deformation and stress in a material. When a material is subjected to changes in temperature, it expands or contracts. This thermal expansion or contraction affects the material's dimensions, leading to changes in shape or size. Thermoelasticity focuses on understanding and characterizing these changes and their influence on the material's mechanical properties. The fundamental principle underlying thermoelasticity is that as the temperature of a material changes, the atomic or molecular constituents within the material also experience thermal motions. These thermal motions result in changes in the interatomic or intermolecular forces, leading to a redistribution of stresses within the material.

Thermoelasticity is governed by two main phenomena:

1. Thermal expansion: When a material is heated, the increase in temperature causes the atoms or molecules within the material to vibrate more vigorously.

This increased vibrational energy leads to an expansion of the material, resulting in changes in its dimensions.

2. Thermal stress: Temperature changes in a material can induce stresses due to the mismatch in the coefficients of thermal expansion between different regions of the material. This stress distribution can cause the material to deform or exhibit structural changes.

Generalized thermoelasticity is a theoretical framework that extends classical continuum mechanics and thermodynamics to describe the behavior of materials that exhibit coupled thermal and mechanical responses. It provides a more comprehensive understanding of the interaction between thermal and mechanical effects in solids. In classical thermodynamics, heat conduction is described by Fourier's law, which assumes that heat flux is directly proportional to the temperature gradient. However, this assumption may not hold for certain materials and conditions. Generalized thermoelasticity takes into account the time delay in the heat conduction process and includes additional mechanisms such as relaxation effects and memory-dependent behaviors. The field of generalized thermoelasticity encompasses various models and approaches, but they generally involve introducing additional constitutive equations or coupling parameters to account for the coupling between temperature and stress fields.

These models can be classified into two main categories: local and non-local.

1. In local models, the governing equations involve partial differential equations with additional constitutive relations that relate the stress, strain, temperature, and their gradients. These models assume that the thermal and mechanical effects are instantaneously transmitted throughout the material without any spatial memory.

2. On the other hand, non-local models consider the existence of spatial memory effects in the material response. These models incorporate integral or differential operators to describe the non-local coupling between different points in the material. The non-local nature of these models allows for the consideration of phenomena such as heat wave propagation and thermal relaxation.

Fractional-order thermoelasticity is a field of study that extends classical thermoelasticity by incorporating fractional derivatives in the governing equations. Thermoelasticity deals with the coupled behavior of temperature and mechanical stress

in materials. In classical thermoelasticity, the governing equations are typically described by partial differential equations that relate the temperature distribution and the displacement field. These equations are based on the assumption of integer-order derivatives, which imply instantaneous and infinitely fast response of the material to changes in temperature or stress. Fractional order thermoelasticity relaxes the assumption of integer-order derivatives and allows for the consideration of non-local and memory-dependent effects in the material response. Instead of integer-order derivatives, fractional derivatives are used, which describe the rate of change of a quantity with respect to time or space in a non-integer order. The fractional order derivatives introduce a memory effect, meaning that the response of the material depends not only on the current state but also on its past history. This enables the modeling of materials that exhibit non-local or hereditary properties, such as viscoelastic materials or materials with long-range interactions.

Origin of the research Problem:

In 1956, Biot M. A. [1] formulated the theory of coupled thermoelasticity to eliminate the paradox inherent in the classical uncoupled theory that elastic changes have no effect on the temperature. The heat equations for both theories are of the diffusion type predicting infinite speeds of propagation for heat waves contrary to physical observations.

In 1967, Lord and Shulman [2] introduced the theory of generalized thermoelasticity with one relaxation time for the special case of an isotropic body. This theory was extended by Dhaliwal and Sherief [3] to include the anisotropic case. In this theory a modified law of heat conduction including both the heat flux and its time derivative replaces the conventional Fourier's law. The heat equation associated with this theory is hyperbolic and hence eliminates the paradox of infinite speeds of propagation inherent in both the uncoupled and the coupled theories of thermoelasticity. The second generalization to the coupled theory of elasticity is known as the theory of thermoelasticity with two relaxation times or the theory of temperature-ratedependent thermoelasticity. In 1969, Fox [4] introduced the generalisation of thermoelasticity is proposed which arises from a physically motivated modification of Fourier's law of heat conduction. Constitutive equations are postulated which are valid for finite deformations and temperature variations and these are reduced to canonical form by the usual techniques of non-linear continuum mechanics.

In 1972, Green and Lindsay [5] developed the theory of generalized thermoelasticity with two relaxation times which is based on a generalized inequality of thermodynamics. This theory does not violate Fourier's law of heat conduction when the body under consideration has a center of symmetry. In this theory both the equations of motion and of heat conduction are hyperbolic but the equation of motion is modified and differs from that of coupled thermoelasticity theory. Müller [6], in review of the thermodynamics of thermoelastic solids, proposed an entropy production inequality, with the help of which the considered restrictions on a class of constitutive equations. Right now there are five different generalizations of the coupled theory of thermoelasticity the details can be found in Hetnarski and Ignaczak [7]. All five theories are based on assumptions of one kind or another. Also, all these theories model the problem of heat conduction in solids as a purely wave propagation phenomenon.

2. Review of Literature and Development in the subject (Previous work done in the related area):

In the last few years, fractional calculus theory has been employed successfully in theories of thermoelasticity and several models of fractional order generalized thermoelasticity are established by many authors.

In 2004, Povstenko [8] made a survey of non-local generalization of the Fourier's law and presented a problem of thermoelasticity that uses fractional heat conduction equation. In 2008, Povstenko [9] solved the time-fractional radial diffusion in a sphere. The Caputo fractional derivative of the order $0 < \alpha < 2$ is used. The Laplace and finite sin-Fourier transforms are employed. The solution is written in terms of the Mittag–Leffler functions. In 2009, Povstenko [10] discussed heat conduction problem with time and space fractional derivatives for the classical Fourier law generalizations. In 2010, Povstenko [11] investigated thermal stresses in the framework of a quasistatic uncoupled theory of thermoelasticity based on heat conduction equation with a time fractional derivative in an infinite medium.

In 2010, Sherief et. al. [12] introduced the fractional order theory of thermoelasticity by using the methodology of fractional calculus, proved uniqueness theorem and derived variational principle and reciprocity theorem. In 2010, Youssef [13] formulated the theory of fractional order generalized thermoelasticity by introducing the Riemann-Liouville fractional integral operator into the generalized heat conduction equation,

proved uniqueness theorem and solved one dimensional problem. In 2011, Ezzat [14] constructed a new mathematical model of thermoelectric modification of the heat-conduction equation from diffusive theory in the context of a new consideration of heat conduction with fractional orders. In 2012, Sur and Kanoria [15] developed a new theory of two-temperature generalized thermoelasticity in the context of a new consideration of heat conduction with fractional orders. In 2014, Raslan [16] investigate the fractional order theory of thermoelasticity to a 1D problem of an infinitely long cylindrical cavity. In 2015, Caputo and Fabrizio [17] developed the new definition of fractional derivative with a smooth kernel which takes on two different representations for the temporal and spatial variable.

In 2016, Tripathi et. al. [18] determining the thermoelastic displacement, stress, and temperature in a thick circular plate of finite thickness and infinite extent whose lower and upper surfaces are traction free, subjected to a given axisymmetric temperature distribution. The problem is formulated in the context of fractional order thermoelasticity theory with finite wave speeds. In 2018, Povstenko and Kyrylych [19] deals the symmetric stress distribution in an infinite isotropic plane with a line crack, in which surfaces are exposed to the heat flux loading is considered in the framework of fractional thermoelasticity. In 2019, Mittal and Kulkarni [20] deals with an application of fractional order thermoelasticity in association with two-temperature theory. The fractional heat conduction model has been proposed to investigate the thermal variations within the bounded spherical region. In 2021, Gaikwad et. al. [21] proposed the fractional order thermoelastic problem for finite piezoelectric rod subjected to different types of thermal loading-direct. approach. In 2022, Gaikwad et. al. [22] studied the two-dimensional generalized magneto-thermo-viscoelasticity problem for a spherical cavity with one relaxation time using fractional derivative. In 2023, Sherief and Hussein [23] proposed the fractional order model of micropolar thermoelasticity and 2D half-space problem.

3. Objectives of Research / Proposed Hypothesis:

The main objectives of this study are

1. To develop mathematical models and equations that accurately describe heat conduction processes that exhibit non-Fourier behavior. Non-Fourier heat conduction models take into account the finite speed of heat propagation, as opposed to the classical Fourier's law, which assumes instantaneous heat transfer.

2. To incorporate non-local effects into the analysis of thermomechanical phenomena. Non-local effects account for the influence of the entire material domain rather than just the local vicinity of a point, which is the assumption made in classical elasticity theory. By considering non-local effects, generalized thermoelasticity provides a more comprehensive description of the behavior of materials under thermal and mechanical loading.

3. To explore the coupling between thermal and mechanical fields. This coupling arises due to the interaction between temperature changes and mechanical deformations in materials. By studying the thermal and mechanical coupling, generalized thermoelasticity provides insights into phenomena such as thermal expansion, thermal stresses, and thermally induced deformations.

4. To analyze wave propagation phenomena in materials. Waves can arise from thermal and mechanical disturbances and can exhibit complex behavior due to non-Fourier heat conduction and non-local effects. Understanding wave propagation in generalized thermoelastic materials is crucial for various applications, including heat transfer, materials characterization, and structural analysis.

5. To develop computational models and numerical techniques that can accurately simulate and predict the behavior of materials under thermomechanical loading. These models and techniques often involve solving coupled partial differential equations that describe the evolution of temperature, displacement, and other relevant fields. The development of efficient and accurate computational methods is essential for practical applications and engineering design.

4. Methodology to be adopted:

1. Formulate the problem: Clearly define the physical problem and establish the boundary and initial conditions. Identify the material properties and assumptions, such as the constitutive equations governing the mechanical behavior and the heat conduction equation.

2. Governing equations: Write down the governing equations of generalized thermoelasticity. These typically include the equations of heat conduction, such as the heat equation, and the equations of linear elasticity, such as the equations of motion or equilibrium equations.

3. Coupling conditions: Specify the coupling conditions that relate the mechanical and thermal fields. These conditions describe how temperature changes affect mechanical deformation and vice versa. Examples of coupling conditions include the thermoelastic coupling coefficient and the thermal expansion coefficient.

4. Analytical or numerical solution: Depending on the complexity of the problem and available resources, you can choose to solve the equations analytically or numerically. Analytical methods involve applying mathematical techniques, such as separation of variables or integral transforms, to obtain closed-form solutions. Numerical methods, such as finite difference, finite element, or boundary element methods, are often employed for more complex geometries and boundary conditions.

5. Solve the equations: Apply the chosen numerical method to solve the system of coupled equations. This typically involves iterating through time or performing a time-stepping procedure to advance the solution in time.

6. Use computational software viz. Maple, Mathematica, MathCAD, MATLAB etc.

7. Validation and verification: Validate the solution by comparing it with known analytical solutions or experimental data whenever possible. Perform sensitivity analyses to investigate the influence of various parameters on the results.

8. Refinement and convergence: If necessary, refine the mesh or adjust the numerical parameters to improve the accuracy and convergence of the solution.

9. Interpretation and discussion: Finally, interpret the results in the context of the original problem. Discuss the implications of the findings and draw conclusions about the behavior of the system under consideration.

5. Importance of study/ society application:

Interdisciplinary Relevance:

Generalized thermoelasticity is an interdisciplinary field that combines principles from thermodynamics, solid mechanics, and heat transfer to study the behavior of materials under the influence of both thermal and mechanical loads. It has several interdisciplinary relevance and applications in various fields.

1. Materials Science and Engineering: Generalized thermoelasticity plays a crucial role in understanding the behavior of materials, particularly in high-temperature environments. It provides insights into the thermomechanical response of materials, allowing for the design of advanced materials with improved thermal stability and mechanical properties.

2. Aerospace Engineering: In aerospace applications, such as the design of aircraft structures and propulsion systems, understanding the thermomechanical behavior of materials is critical. Generalized thermoelasticity helps engineers analyze and predict the response of components under extreme thermal and mechanical loading conditions, ensuring the structural integrity and safety of aerospace systems.

3. Energy Systems: Generalized thermoelasticity is relevant in the analysis of energy systems, such as power plants and thermal energy storage systems. It enables engineers to model and optimize the performance of components subjected to cyclic thermal and mechanical loading, leading to enhanced efficiency and reliability of energy systems.

4. Biomechanics: The human body is subjected to both thermal and mechanical loads in various physiological processes and medical applications. Generalized thermoelasticity helps researchers and clinicians understand the thermomechanical behavior of biological tissues, such as bones, muscles, and blood vessels. This knowledge is essential for designing medical devices, evaluating the effects of thermal therapies, and studying the mechanics of biological systems.

5. Microelectronics and Nanotechnology: In the design and analysis of microelectronic devices, understanding the thermomechanical response of materials is crucial for reliable performance. Generalized thermoelasticity provides insights into the thermal expansion, stress generation, and heat dissipation in microelectronics, enabling engineers to optimize device performance, prevent failure, and improve the overall reliability of electronic systems.

6. Geophysics: Generalized thermoelasticity is relevant in geophysical studies, particularly in understanding the behavior of rocks and Earth's crust under thermal and mechanical loads. It helps geophysicists model processes such as rock deformation, earthquakes, and thermal convection, contributing to a better understanding of Earth's dynamics and the development of predictive models.

International & National Status:

At the international level, thermoelasticity has been an active area of research, with contributions from scientists and engineers around the world. Researchers have developed various theoretical models and computational techniques to study the behavior of materials under thermal and mechanical loads. The applications of thermoelasticity can be found in diverse fields, including engineering, materials science, aerospace, and geophysics.

In terms of national status, the extent of research and development in thermoelasticity can vary among countries. Countries with strong research institutions and advanced engineering industries are likely to have active research groups and ongoing studies in thermoelasticity.

Significance of the study:

Generalized thermoelasticity is a branch of solid mechanics that extends classical thermoelasticity to account for the effects of non-Fourier heat conduction and the coupling between thermal and mechanical fields. It is a more comprehensive and realistic approach to modeling the behavior of materials under thermal and mechanical loads. Here are some significant aspects of generalized thermoelasticity:

1. Non-Fourier heat conduction: Classical thermoelasticity assumes that heat conduction follows Fourier's law, which states that heat flux is proportional to the temperature gradient. However, in certain situations, such as at small length and time scales or in materials with high thermal gradients, the assumption of Fourier's law breaks down. Generalized thermoelasticity introduces alternative heat conduction models, such as the hyperbolic or the dual-phase-lag equations, which can capture the non-Fourier effects accurately.

2. Finite speed of heat propagation: Fourier's law implies an infinite speed of heat propagation, which is physically unrealistic. In contrast, generalized thermoelasticity allows for finite propagation speeds of thermal disturbances. This is particularly important in situations involving rapid thermal processes or in materials with significant thermal inertia.

3. Thermo-mechanical coupling: Generalized thermoelasticity considers the coupling between the thermal and mechanical fields. Temperature changes can induce mechanical strains, and mechanical deformations can generate temperature variations through the adiabatic or dissipation processes. By accounting for this coupling, generalized thermoelasticity provides a more accurate description of the material response under simultaneous thermal and mechanical loads.

4. Wave propagation phenomena: Generalized thermoelasticity predicts wave phenomena that are not captured by classical thermoelasticity. For example, the dual-phase-lag model can exhibit wave dispersion, in which different frequencies of thermal disturbances propagate at different speeds. This phenomenon can have implications in various fields, including heat transfer, materials processing, and structural dynamics.

5. Improved accuracy and predictive capabilities: By incorporating non-Fourier heat conduction and thermo-mechanical coupling, generalized thermoelasticity can provide more accurate predictions of temperature distribution, stress, and strain fields in materials subjected to thermal and mechanical loading conditions. This enhanced accuracy can be crucial in designing and analyzing systems where thermal effects play a significant role.

6. Proposed work Plan / Formulation & Structure of Study:

Year wise plan of work and targets to be achieved are as follows.

1. **First Year:**

   - Attend the seminars/workshops on relevant topic.
   - Gain a thorough understanding of the fundamentals of generalized thermoelasticity.
   - Complete a comprehensive literature review on the topic.
   - Develop a computational or analytical model for simulating thermoelastic behavior in generalized thermoelastic materials.
   - Perform preliminary simulations or analytical studies to validate the model.

2. **Second Year:**

   - To attend the seminars/conference/workshops on relevant topic.
   - Perform extensive simulations or analytical investigations on various generalized thermoelastic materials.
   - Develop improved numerical algorithms or analytical techniques for solving the governing equations.
   - Identify discrepancies or limitations in existing theories and propose modifications or extensions.

3. **Third Year:**

   - Develop advanced computational tools or solving techniques for studying complex thermoelastic phenomena.
   - Investigate the behavior of generalized thermoelastic materials under extreme conditions.
   - Study the stability and failure mechanisms of generalized thermoelastic materials.
   - Publish research papers in conferences and reputed journals.
   - To submit the work to the University.

Date: 09/06/2023

Mrs. Sandhya U. Lokhande
(Research Student)

Date: 09/06/2023

Dr. Pramod R. Kulkarni
(Research Guide)

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